CALCULATION OF THE HEATING OF LAYERED BODIES

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An expression is obtained for the temperature nonuniformity parameter for a pack of plates and the conditions for the transition from the pack to a quasiuniform body are analyzed.

Calculation of the heating (cooling) of multicomponent systems under nonstationary conditions is a complicated problem. To simplify the calculation for an inhomogeneous body, the body is replaced by a quasiuniform body with effective values of the thermophysical properties [1]. This assumption led a number of authors to propose approximate methods for calculating nonstationary heat transfer through multilayered shells [2-4].

This work is concerned with the problem of developing simple and accurate methods for calculating the heating of layered bodies (rolls, packs, multilayered pipes, etc.) under conditions such that the thermal conductivity of one component relative to another is high ($\lambda/\lambda_g \gg 1$), while the volume heat capacity of the interlayers is negligibly small ($c_g\rho_g/c\rho \ll 1$). Numerical calculations were used to estimate the errors in the approximate solutions obtained.

We are examining two-sided convective heating of a plane layered body, consisting of N identical plates with thickness δ_m, separated by identical gaps with thickness δ_g. The external plates are heated by a gas flow with coefficients of heat transfer α_1 and α_2 from the medium, with temperature t_{g_1} and t_{g_2}. Each of the plates may be assumed to be a thermally thin body. The system of equations for calculating the temperatures of the plates $t_1, t_2, \ldots, t_i, \ldots, t_N$ has the following form:

$$\begin{gathered}
c\rho\delta_m\frac{dt_1}{d\tau} = \alpha_1(t_{g_1} - t_1) + \beta_1(t_2 - t_1),\\
c\rho\delta_m\frac{dt_i}{d\tau} = \beta_{i-1}(t_{i-1} - t_i) + \beta_i(t_{i+1} - t_i),\\
c\rho\delta_m\frac{dt_N}{d\tau} = \beta_{N-1}(t_{N-1} - t_N) + \alpha_2(t_{g_2} - t_N),
\end{gathered} \tag{1}$$

where i = 2, 3, ..., N − 1;

$$\beta_i = \frac{\lambda_g}{\delta_g} + 4\sigma_0\varepsilon_{red}T_i^3. \tag{2}$$

We solved the system (1) with the help of a computer using the Runge–Kutta method with fixed variation of the temperature of the medium as a function of time.

We shall represent the packet of plates as a continuous body with effective coefficient of thermal conductivity

$$\lambda_{eff} = \frac{\delta_m N}{\dfrac{N\delta_m}{\lambda_m} + \sum\limits_{i=1}^{N-1}\dfrac{1}{\beta_i}} \approx \frac{\delta_m N}{\sum\limits_{i=1}^{N-1}\dfrac{1}{\beta_i}} \tag{3}$$

and a heat capacity which is determined by the material of the plates. This approach is exact in calculating the stationary flow of heat through a pack consisting of thermally thin plates.

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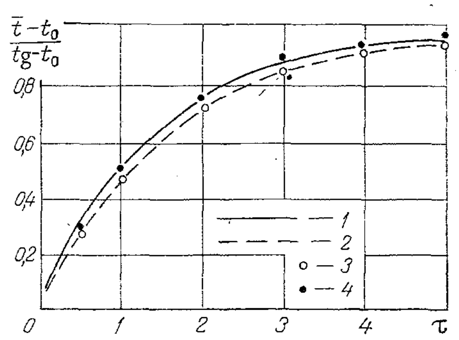

Fig. 1. Results of the determination of the relative mean bulk temperature of a seven-layer pack using different methods of calculation (τ, h): 1) numerical solution; 2) analytical solution according to [5]; 3) using expression (5) with $R_{in} = 1/3$; 4) using expression (5) with Ψ from (11); $t_g = 260°C$; $\alpha = 20\ W/m^2 \cdot K$; $c = 0.5 \cdot 10^3\ J/kg \cdot K$; $\rho = 7.85 \cdot 10^3\ kg/m^3$; $t_0 = 20°C$; $\delta_m = 5.4 \cdot 10^{-3}\ m$; $\lambda_m = 40\ W/m \cdot K$; $\delta_g = 1.0 \cdot 10^{-3}\ m$; $\lambda_g = 0.032\ W/m \cdot K$; $\sigma_0 = 5.7 \cdot 10^{-8}\ W/m^2 \cdot K^4$; $\varepsilon_{red} = 0.667$; $N = 7$.

In calculations of nonstationary heating of the pack, it is natural to expect some error [4].

As an example, Fig. 1 shows the results of a numerical calculation, using Eqs. (1), of the relative average bulk temperature of a pack consisting of seven metallic sheets with symmetrical heating of the pack by convection in a medium with constant temperature (curve 1). The figure also presents the results of an analytical solution [5] for a continuous body using the effective coefficient of thermal conductivity (curve 2). The difference from the numerical calculation is 8% for $Fo \geqslant 0.6$ ($\tau \geqslant 1$ h).

The calculations using the approximate equations, taking into account the internal thermal resistance of the pack by means of the temperature nonuniformity parameter [6, 7]

$$\Psi = (1 + R_{in}\mathrm{Bi})^{-1}, \tag{4}$$

were performed using the equation

$$\bar{t} = t_g - (t_g - t_0)\exp(-A\tau), \tag{5}$$

where $A = 2\alpha\Psi/c\rho\delta_m N$.

Expression (5) was obtained assuming that at some time τ a regular regime of type I appears, as a result of which Ψ does not change with time. For $Bi < 3$ and $R_{in} = 1/3$ for $Fo > 0.5$, the results of calculations using (5) differ considerably from the numerical solution (1), but practically coincide with the calculation of heating with boundary conditions of the third kind (see Fig. 1). The reason for this lies in the fact that under the conditions of nonstationary heating, the pack of plates being studied cannot always be assumed to be a continuous body with a coefficient of thermal conductivity λ_{ef} given by (3) [4].

To derive approximate dependences taking into account the characteristics of the heating of the layered body and to derive estimates of the conditions for reducing it to a quasiuniform body with nonstationary heating, we shall examine the heating of the pack under the conditions of a regular type-II regime, i.e., when

$$\frac{dt_i}{d\tau} = R = \mathrm{const}. \tag{6}$$

Then, from system (1), substituting (6), we obtain for the temperature of the i-th plate (with $t_{g1} = t_{g2} = t_g$; $\alpha_1 = \alpha_2 = \alpha$; $\beta_i = \bar{\beta}$):

$$t_g - t_i = \frac{c\rho\delta_m R}{2\alpha} + \frac{c\rho\delta_m R}{2\bar{\beta}}(i-1)(N-1). \quad (7)$$

The quantity R_{in} with heating of the packet can be determined from the relation

$$c\rho\delta_m N \frac{d\bar{t}}{d\tau} = \frac{2\alpha}{1 + R_{in}\mathrm{Bi}}(t_g - \bar{t}) = 2\alpha(t_g - t_1) \quad (8)$$

or, substituting (7),

$$R_{in} = \frac{\alpha}{N^2\bar{\beta}\,\mathrm{Bi}} \sum_{i=1}^{N} (i-1)(N-i) = \frac{\alpha}{N\bar{\beta}\,\mathrm{Bi}} \frac{(N-1)(N-2)}{6}. \quad (9)$$

We shall write the effective thermal conductivity for the layered body, neglecting the thermal resistance of the metallic sheets, as $\lambda_{ef} = \delta_m N\bar{\beta}/(N-1)$, while the number $\mathrm{Bi} = \alpha(N-1)/2\bar{\beta}$. Then, from (9) we obtain the following expression for R_{in}:

$$R_{in} = (N-2)/3N. \quad (10)$$

It is evident that for $N \to \infty$ $R_{in} \to 1/3$ and, in addition, $\mathrm{Bi} \to \mathrm{const}$, since as the number of plates increases with the thickness of the pack remaining constant and the total thickness of the plates remaining constant, the number of interlayers increases and the thicknesses of separate plates and interlayers decrease.

Using (10), we shall write down an expression for the temperature nonuniformity parameter as applied to a layered body:

$$\Psi_{lay} = \left(1 + \frac{N-2}{3N}\mathrm{Bi}\right)^{-1}. \quad (11)$$

Numerical calculations for the example examined in Fig. 1 gave $R_{in} = 0.236$. The results of a calculation of the average temperature of the pack using Eqs. (5) and (11) are shown in Fig. 1 by curve 4. For $\mathrm{Fo} > 0.6$ ($\tau > 1$ h), the difference from the numerical solution is less than 1.5%.

We shall estimate the error in applying the continuous-body approximation to the layered body by comparing the heating time up to the same average bulk temperature with constant temperature of the heating medium:

$$\frac{\tau_{cont}}{\tau_{lay}} = \frac{1 + \mathrm{Bi}/3}{1 + \frac{N-2}{N}\mathrm{Bi}}.$$

This equation is valid, as is expression (4), for $\mathrm{Bi} < 3$ with boundary conditions of the third kind.

Given the error in determining τ, i.e., $(\tau_{cont} - \tau_{lay})/\tau_{lay} \leqslant \gamma$, we obtain an expression for the minimum number of plates in the pack, below which the computational error will exceed γ:

$$N_{min} = \frac{2(1+\gamma)}{\gamma(1 + 3/\mathrm{Bi})}. \quad (12)$$

From expression (12), we can see the dependence of N_{min} on the number Bi. If the error in determining τ equals 5% ($\gamma = 0.05$), then we find that with $\mathrm{Bi} = 3$, $N_{min} = 21$. For $\mathrm{Bi} \to 0$, $N_{min} \to 0$, i.e., any layered body can be viewed as being quasiuniform for very low intensity of external heat transfer compared with the internal transfer.

We shall estimate the validity of the continuous-body approximation to the layered body under conditions of the regular type-II regime. As a criterion for making comparisons, we shall choose the relative difference in temperatures between the surface and the center of the body in the presence of symmetrical heating of the body. As a result, for an even number of plates in the pack, we obtain $\Delta t_{cont}/\Delta t_{lay} = (N-1)/(N-2)$, while in the case of an odd number of plates $\Delta t_{cont}/\Delta t_{lay} = N/(N-1)$.

If we limit ourselves to an error of 5% in determining the temperature difference in the heated body, then the number of plates in the pack must be N_{min} = 20-21.

The absolute difference in the temperature differentials between the continuous and layered body $\Delta t = (\Delta t_{cont} - \Delta t_{lay})$ is $\Delta t_{even} = q/4\bar{\beta}$ for an even number of plates and $\Delta t_{odd} = \frac{q}{4\bar{\beta}} \cdot \frac{N-1}{N}$ for an odd number of plates.

Analysis of the expressions obtained shows that for β = const and $N \to \infty$, the absolute difference in the temperature differentials does not vanish. When the overall thickness of the body remains the same with an increasing number of plates, $\bar{\beta}$ increases and for $\bar{\beta} \to \infty$, $\Delta t \to 0$.

NOTATION

δ_m, δ_g, thicknesses of the layers of metal and of gas; λ_m, λ_g, coefficients of heat conduction in the metal and in the gas; c, ρ, heat capacity and density of the metal; c_g, ρ_g, heat capacity and density of the gas in the interlayer; β_i, conductivity of the i-th gaseous interlayer; ε_{red}, reduced emissivity in the gap; σ_0, thermal radiation constant; T_i, average temperature in the gap; t_0, starting average temperature of the pack; t_g, temperature of the medium; $\bar{t}$, average temperature of the body; α, coefficient of heat transfer from the surrounding medium to the body; R_{in}, internal thermal resistance of the body; Ψ, a parameter describing the nonuniformity of the temperature field; τ, time; R, rate of heating of the metal; τ_{cont}, τ_{lay}, heating times of the continuous and layered bodies; λ_{ef}, the effective coefficient of thermal conductivity; N, temperature differentials between the surface and the center of the continuous and layered bodies; Δt_{cont}, Δt_{lay}, absolute differences in the temperature differentials between the continuous and layered bodies; Δt_{even}, Δt_{odd}, absolute differences in the temperature differentials between the continuous and layered bodies as a whole, with even and odd numbers of plates in the pack; q, heat flux to the surface of the body; $Bi = \alpha\delta_m N/2\lambda_{ef}$; and, $Fo = 4\lambda_{ef}\tau/c\rho\,(\delta_m N)^2$.

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